Entropy generation in a rectangular packed duct with wall heat flux

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Abstract

The entropy generation due to heat transfer and friction has been calculated for fully developed, forced convection flow in a large rectangular duct, packed with spherical particles, with constant heat fluxes applied to both the top (heated) and bottom (cooled) wall. An approximate analytical expression for the velocity profile developed for packed bed with $H/d_p > 5$ has been used together with the energy equation of fully developed flow to calculate the non-isothermal temperature profiles along the flow passage. The velocity profile takes into account the increase in the velocity near the wall due to the higher voidage in this region of the bed. The effect of the asymmetric heating on the velocity profile is neglected under the thermal conditions considered. The volumetric entropy generation rate and the irreversibility distribution ratios have been calculated and displayed graphically for the values of $H/d_p = 5$ and 20. It was found that the irreversibility distributions are not continuous through the wall and core regions, hence the optimality criterion of equipartition of entropy generations should be considered separately for these regions of the packed duct. © 1999 Elsevier Science Ltd. All rights reserved.

Nomenclature

A area [m^2]
Be Bejan number
C_p heat capacity [J kg^{-1} °C^{-1}]
d_p diameter of packing [m]
h average heat transfer coefficient [W m^{-2} K^{-1}]
H depth of duct [m]
k_e effective thermal conductivity of fluid [W m^{-1} K^{-1}]
k_f thermal conductivity of fluid [W m^{-1} K^{-1}]
N $S_{gen}''' \dfrac{k_e T_0^2}{Q^2}$, dimensionless
Nu Nusselt number, $Nu = h d_p / k_f$
Q heat flux rate [W m^{-2}]
r Q_2/Q_1
Re_p Reynolds number, $Re_p = u_{av}\rho d_p/\mu$
Re_H Reynolds number, $Re_H = u_{av}\rho H/\mu$
S''' volumetric rate of entropy generation [W m^{-3} K^{-1}]
St Stanton number, $St = h/\rho u_{av} c_p$
T temperature [K]
u velocity [m s^{-1}]
W width of duct [m]
x direction of fluid flow
X dimensionless coordinate, x/H
y direction normal to the flow direction
Y dimensionless coordinate, y/H.

Greek symbols

α_e effective thermal diffusivity [m^2 s^{-1}]
ε void fraction
μ Newtonian fluid viscosity [kg m^{-1} s^{-1}]
ρ density [kg m^{-3}]
ϕ irreversibility distribution ratio
τ entrance temperature number, $\tau = Q_1/(hT_0)$.

Subscripts

b   bulk
cal   calculated
e   effective, equivalent
exp   experimental
f   fluid
p   packing
w   wall

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ΔP finite pressure
ΔT finite temperature.

1. Introduction

Fluid flow and wall-to-fluid heat transfer in a packed tube or channel are of interest for several applications such as chemical fixed bed reactors, heat exchangers and cooling of electronic equipment [1]. Demirel [2], Hwang et al. [3] and Abu-Al-Saud [4] reported that the wall-to-gas heat transfer in a rectangular duct with asymmetric heating increased by a factor of three after introducing packing into the flow passage. The combined utilization of the first and second laws of thermodynamics yields a new approach to analyze the asymmetrically heated packed bed based on the net entropy generation [5]. Recently, Demirel and Al-Ali [6] and Demirel et al. [7] investigated the volumetric entropy generation rate due to convection heat transfer and friction in a rectangular packed duct with asymmetric heating, and they showed that packing causes equipartition of entropy generation over the cross section of the bed, yielding an optimum transport in the sense of thermodynamics [8, 9]. In the previous studies [6, 7], a plug-type profile was assumed for the velocity. However when large particles are present in the bed, the wall region affects the flow substantially. The properties of the wall region are directly related to H/d_p, especially for gas-phase systems, and some published works emphasize the influence of velocity distortions near the wall, mainly due to porosity change, on the performance of packed beds [10–13].

In the present study the wall region effect on the velocity profile has been taken into account in the calculations of the entropy generation in a packed duct with the top wall heated and bottom wall cooled. An approximate analytical expression for the wall velocity profile in a packed bed with $H/d_p > 5$ given by Vortmeyer and Schuster [14] was modified to cover the whole separation distance H. This velocity profile is inserted to the fully developed energy equation to calculate the spatial temperature profiles when H/d_p varies between 5 and 20. The temperature and velocity gradients thus derived allow the calculation of the entropy generation due to heat transfer and friction. It was found that the wall region adversely affects the optimality criterion [8, 9] of equipartition of irreversibility distributions in the packed bed for a required transport load. The effect of asymmetric wall temperature on the flow velocity profiles is neglected since maximum temperature differences between the walls are kept small.

2. Irreversibility distributions

The rectangular duct with asymmetric heating is shown in Fig. 1. It is assumed that the difference of wall-to-fluid
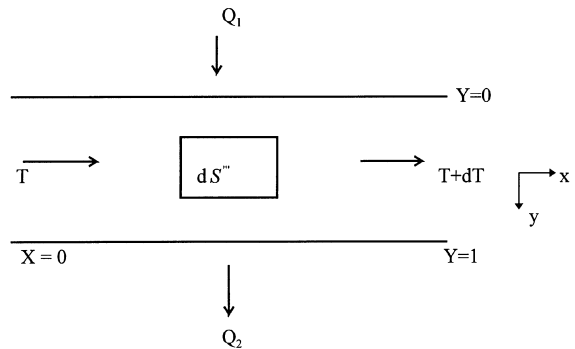

Fig. 1. Control volume of the rectangular packed bed with asymmetric heat effects.

bulk temperature is small enough, and that there is no considerable change in physical properties of the fluid, no axial conduction, and no natural convection.

An approximate analytical expression for the flow profile near the wall in a packed bed with $H/d_p > 5$ at 293 K is given by Vortmeyer and Schuster [14]

$$\frac{u}{u_b} = \left[1 - \left(1 - n\frac{y}{d_p}\right)e^{a\frac{y}{d_p}}\right] \quad (1)$$

where y is the distance from the top wall and u_b is the bulk velocity. The parameters n and a are given as

$$n = -1803 + 201.6(\ln Re_p + 4) - 3737(\ln Re_p + 4)^{0.5}$$
$$+ 5399(\ln Re_p + 4)^{1/3} \quad 1 < Re_p < 1000 \quad (2)$$

$$a = \frac{4n}{4 - n} \quad (3)$$

where

$$Re_p = \frac{u_{av}\rho d_p}{\mu}$$

Using the term $(H - y)$ in place of y in equation (1), the velocity profile near the bottom wall can be obtained

$$\frac{u}{u_b} = \left[1 - \left(1 - n\frac{H - y}{d_p}\right)e^{a\frac{(H-y)}{d_p}}\right] \quad (4)$$

The velocity profile for the whole region $(0 < y < H)$ can be obtained by adding equations (1) and (4) and subtracting unity. The bulk velocity can be obtained by averaging the velocity u over the separation distance H, and the following expression is found for u in terms of the average velocity u_{av}

$$U = \frac{u}{u_{av}} = \beta\{1 - (1 - a_3 Y)e^{a_1 Y} - [1 - a_3(1 - Y)]e^{a_1(1-Y)}\}$$

$$(5)$$

The term β denotes the deformation factor that depends on the H/d_p and Re_p, and can be expressed by

$$\beta = \left(1 + \frac{2 - 2a_2 + 2a_2a_3}{a_1} + \frac{2a_3(1 - a_2)}{a_1^2}\right)^{-1} \quad (6)$$

where

$$a_1 = aH/d_p, \quad a_2 = e^{a_1}, \quad a_3 = nH/d_p, \quad Y = y/H \quad (7)$$

The values of β approach unity as H/d_p approaches infinity. The velocity profile peaks in the vicinity of the wall while the velocity in the core region attains a lower and roughly constant value. Figure 2(a) and (b) shows the flow velocity profiles for the $H/d_p = 5$ and 10, respectively. The effect of deformation on the velocity profile is clearly seen with increasing values of H/d_p. The velocity gradient is obtained as

$$\frac{\mathrm{d}u}{\mathrm{d}y} = \frac{\beta u_{av}}{H}[(a_1a_3Y - a_1 + a_3)e^{a_1Y}$$
$$+ (a_1a_3(1 - Y) + a_1 - a_3)e^{a_1(1 - Y)}] \quad (8)$$

For a constant heat flux, a simple energy balance ($Q\,\mathrm{d}x = \rho u_{av}HC_p\,\mathrm{d}T_b$) yields the temperature gradient in the flow direction

(a)

U

Y

(b)

U

Y

Fig. 2. Velocity profiles in the packed bed for: (a) $H/d_p = 5$, $Re_p = 309$, $\beta = 0.816$; (b) $H/d_p = 10$, $Re_p = 154$, $\beta = 0.871$.

$$\frac{\partial T}{\partial x} = \frac{\mathrm{d}T_b}{\mathrm{d}x} = \frac{Q_1(1 - r)St}{hH} = \text{constant} \quad (9)$$

where St is the Stanton number, h is the average heat transfer coefficient and $r = Q_2/Q_1$. The energy equation for a fully established laminar flow is [15, 16]

$$\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) = \frac{u}{\alpha_e}\left(\frac{\mathrm{d}T_b}{\mathrm{d}x}\right) \quad (10)$$

where α_e is the effective thermal diffusivity. The heat flux at the upper and lower surface (Fig. 1) specifies the temperature gradient at the wall, thus providing the necessary boundary conditions:

at $y = 0 \quad -k_e(\partial T/\partial y) = Q_1 = \text{constant}$

at $y = H \quad -k_e(\partial T/\partial y) = Q_2 = \text{constant}$

The linearity of the energy equation suggests that the superposition method may be employed to build solutions for asymmetric heating by adding the two fundamental solutions: (1) the top wall heated with the other insulated ($\partial T/\partial y = 0$) and (2) the bottom wall cooled with the other insulated. The fundamental solutions can be obtained by inserting the velocity profile, given by equation (5), into the energy equation (10), and are given by

$$T^{(1)} - T_0 = \frac{\beta Q_1 H}{k_e}[B + 0.5Y^2 - CY - D] + \frac{Q_1}{h}(1 + StX) \quad (11)$$

$$T^{(2)} - T_0 = -\frac{\beta Q_1 H}{k_e}[B + 0.5Y^2 - EY - G]$$
$$+ \frac{rQ_1}{h}(1 + StX) \quad (12)$$

where

$$X = x/H$$

$$B = \left[\frac{a_3Y(e^{a_1Y} - a_2e^{-a_1Y})}{a_1^2}\right.$$
$$\left. - \frac{(a_1 + 2a_3)e^{a_1Y} - a_2(a_1a_3 - a_1 - 2a_3)e^{-a_1Y}}{a_1^3}\right]$$

$$C = [a_1(a_1 + 1 + a_2a_3 - a_2) + a_3(1 - a_2)]/a_1^2$$

$$D = [a_1(a_2a_3 - a_2 - 1) - 2a_3(a_2 + 1)]/a_1^3$$

$$E = [a_2(a_3 - a_1a_3 + a_1) - a_3 - a_1]/a_1^2$$

$$G = [a_1(a_2a_3 - a_2 - 1) - 2a_3(1 + a_2)]/a_1^3 + 0.5 - E$$

The temperature profile for the rectangular packed bed with asymmetric heating can be obtained by adding the two fundamental solutions of equations (11) and (12), and is given by
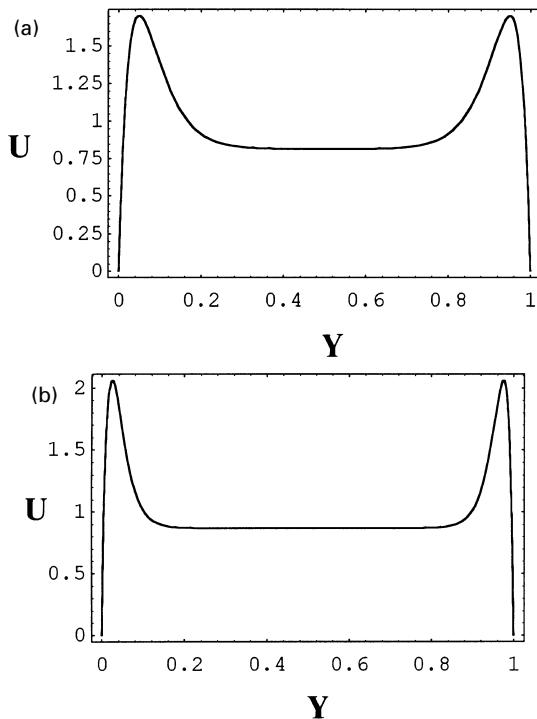
$$T = T_0(1 + \tau\psi) \tag{13}$$

where

$$\tau = \frac{Q_1}{hT_0}$$

$$\psi = \frac{\beta hH}{k_e}[(1-r)(B+0.5Y^2)$$

$$+(rE-C)Y + rG - D] + (1-r)(StX+1)$$

The bulk temperature is expressed by

$$T_b(x) = \frac{\int_A uT\,dA}{\int_A u\,dA} \tag{14}$$

The local bulk temperature calculations are tested with experimental data [4] obtained from a rectangular, sphere packed duct with heated top wall at $H/d_p = 2.08$, as shown in Fig. 3. The agreement between the calculated and experimental temperatures is satisfactory. The temperature profile, given by equation (13), is shown in Fig. 4 for the thermal boundary condition of $r = 0.2$ in a packed bed with $H/d_p = 10$.

The effective thermal conductivity is taken from Kulkarni and Doraiswamy [17]

$$k_e = k_f\left[\varepsilon + (1-\varepsilon)\left(0.2 + \frac{2k_f}{3k_p}\right)^{-1}\right] + Re_p\left(\frac{0.0025}{1+46(d_p/d_e)}\right) \tag{15}$$

where d_e is the equivalent diameter of the bed, and is given by Perry [18]

$$d_e = [3.05H^2W^2/(H+W)]^{1/3}$$

The void fraction decreases with distance from the wall. Vortmeyer and Schuster [14] used the following exponential expression for ε

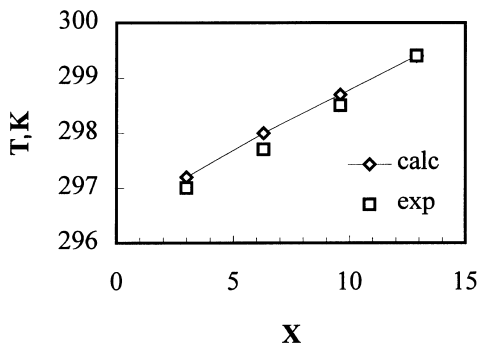$$\varepsilon = \varepsilon_0\left[1 + \lambda\exp\left(1 - 2\frac{y}{d_p}\right)^3\right] \tag{16}$$

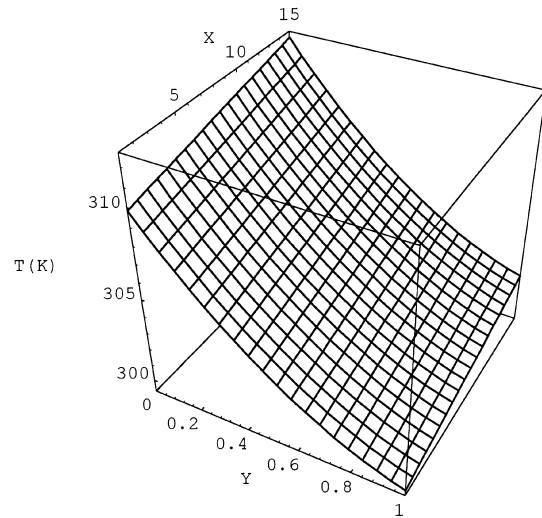Fig. 3. Local bulk temperatures in the flow direction in the packed bed.

Fig. 4. Temperature profile in the packed bed for $H/d_p = 10$, $Q_1 = 66$ W m^{-2}, $r = 0.2$, $Re_p = 154$, $\beta = 0.871$.

for a rectangular bed where λ is adjusted according to ε_0 [14]. The void fraction ε_0 may be approximated by the following equation given for spherical packing in a cylindrical bed [19].

$$\varepsilon_0 = 0.383 + 0.25\left(\frac{d_e}{d_p}\right)^{-0.923}\left(0.723\frac{d_e}{d_p} - 1\right)^{-1/2}$$

$$d_e/d_p > 1.89 \tag{17}$$

Temperature gradient in the y-direction may be obtained from equation (13) and given by

$$\frac{\partial T}{\partial y} = \frac{\beta Q_1}{k_e}[(1-r)(B^* + Y) + rE - C] \tag{18}$$

where

$$B^* = \left[\frac{a_3Y(e^{a_1Y} - a_2e^{-a_1Y})}{a_1}\right.$$

$$\left. - \frac{(a_1+a_3)e^{a_1Y} - a_2(a_1a_3 - a_1 - a_3)e^{-a_1Y}}{a_1^2}\right]$$

The Gouy–Stodola theorem states that the lost available energy in a convection heat transfer is directly proportional to the entropy generation. The volumetric rate of entropy generation for the packed duct under consideration is given by [5]

$$S''' = \frac{k_e}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T}\left(\frac{du}{dy}\right)^2 \tag{19}$$

Here the first term on the right-hand side shows the entropy generated due to heat transfer, $S'''_{\Delta T}$, while the entropy generated due to fluid friction, $S'''_{\Delta P}$, is shown by the second term. Equation (19) expresses the volumetric entropy generation rate in terms of H/d_p, heat duty, Q, the Reynolds number and the Stanton number. The irreversibility distribution ratio ϕ is defined as

$$\phi = S'''_{\Delta P}/S'''_{\Delta T} \tag{20}$$

An alternative irreversibility distribution parameter called the Bejan number, *Be*, is given by [20]
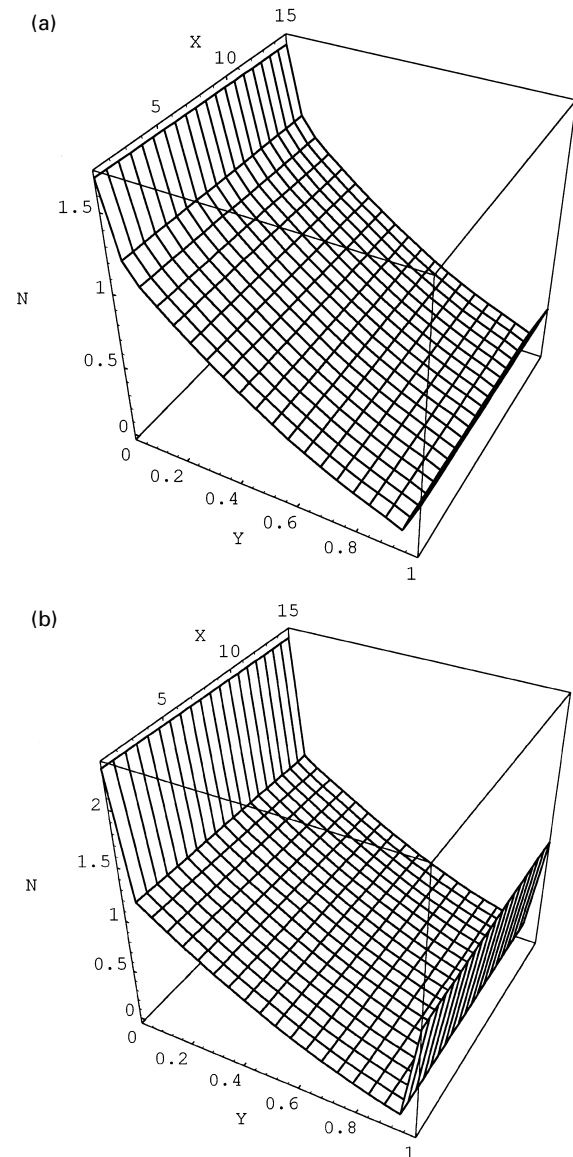
Fig. 5. Distributions of entropy generation, *N*, in the packed bed: (a) $H/d_p = 10$, $Q_1 = 66$ W m^{-2}, $r = 0.2$, $Re_p = 154$, $\beta = 0.871$; (b) $H/d_p = 20$, $Q_1 = 66$ W m^{-2}, $r = 0.2$, $Re_p = 77$, $\beta = 0.905$.
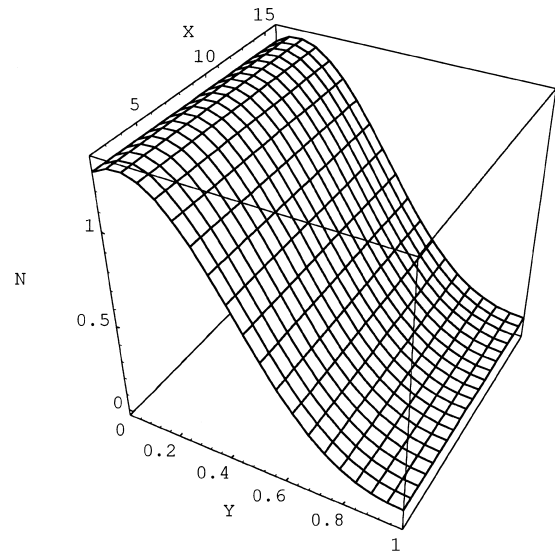

Fig. 6. Distributions of entropy generation, *N*, profile in the empty bed for $Re_H = 1641$, $St = 0.005$, $Q_1 = 34$ W m^{-2}, $r = 0.2$.
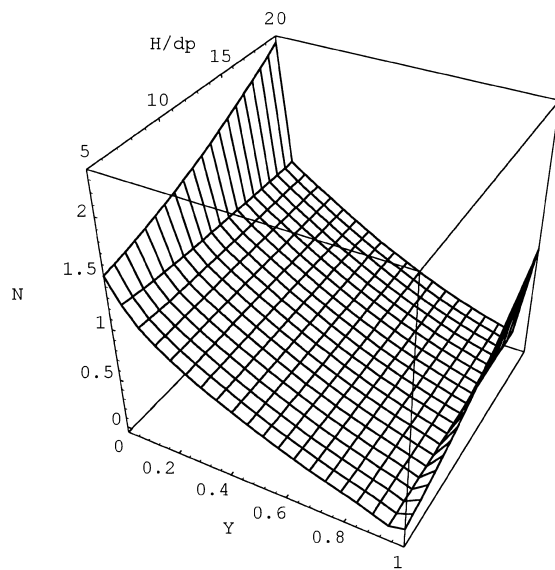

Fig. 7. Distribution of entropy generation with *Y* and H/d_p at $X = 10$.

$$Be = S'''_{\Delta T}/S''' = 1/(1+\phi) \tag{21}$$

At *Be* = 1, all the irreversibility is due to heat transfer. When $Be > \frac{1}{2}$, the irreversibility due to heat transfer dominates. After inserting the velocity and temperature gradients given by equations (8), (9) and (18), the volumetric entropy generation can be evaluated. The non-dimensional volumetric entropy generation *N* can be obtained as

$$N = S''' \frac{k_e T_0^2}{(Q_1 - Q_2)^2} \qquad (22)$$

3. Results and discussion

The following constant air properties are used: $k = 0.026$ W m^{-1} K^{-1}, $\mu = 1.84$ 10^{-5} kg m^{-1} s^{-1}, $\rho = 1.17$ kg m^{-3}, $Pr = 0.7$. They are calculated for the inlet air flow temperature $T_0 = 297$ K. Y changes between 0 and 1, while X varies between 2 and 16 for the fully developed flow region of the bed. The asymmetric thermal boundary condition considered is $r = 0.2$ with small heat fluxes ($Q_1 = 66$ W m^{-2}), since the non-isothermal effect on the velocity profile is neglected. The Nusselt number for a fully established flow between the empty parallel plates is given by [15]

$$Nu = \frac{5.385}{1 - 0.346r} \quad (r < 2.9) \qquad (23)$$

It was assumed that the average heat transfer coefficient is about three times higher than that given by equation (23) after Demirel [2], Hwang et al. [3] and Abu-Al-Saud [4], when the spherical packing is introduced into the air flow passage.

Figure 5 shows the dimensionless entropy generation rate in the packed bed for two values of H/d_p at $r = 0.2$. Figure 5(a) shows the map of N for $Re_p = 154$, $St = 0.016$, $Q_1 = 66$ W m^{-2}, $\varepsilon = 0.386$, $H/d_p = 10$, $\beta = 0.871$, $n = 22.48$. Non-uniformities in the entropy generation rate are visible at around $Y = 0.1$ and $Y = 0.9$ corresponding to the wall regions of the heated and cooled walls, respectively. At a higher value of $H/d_p = 20$, the distribution of N becomes more distorted in the wall regions that are relatively narrower. Only the region away from the wall effect, where a uniform velocity profile prevails, shows a uniform distribution of entropy generation in the packed bed. This is in line with Demirel and Al-Ali [6] and Demirel et al. [7]. Obviously the narrowing wall region at high values of H/d_p causes relatively higher flow velocities that yield sharp changes in the entropy generation.

When there is no packing in the bed, the temperature profiles can be calculated similarly using the parabolic velocity profile of $u = 6u_{av}(Y - Y^2)$ with the superposition approach and the following temperature profile is obtained for a rectangular bed with asymmetric heating:

$$T^* = T_0(1 - \tau \psi^*) \qquad (24)$$

where

$$\psi^* = \frac{hH}{2k_f}[(r-1)(Y^4 - 2Y^3 - 2Y + r)] + (1-r)(StX + 1)$$

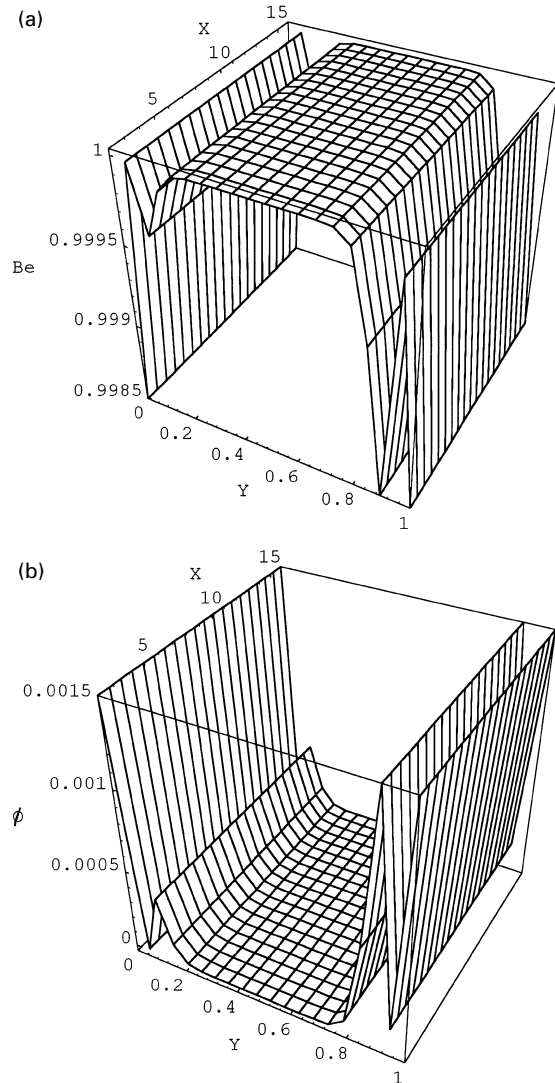Using the gradients of velocity and temperature in equa-

Fig. 8. (a) Bejan number *Be*; (b) irreversibility distribution ratio ϕ; for $H/d_p = 5$, $Re_p = 309$, $\beta = 0.816$, $Q_1 = 66$ W m^{-2}, $r = 0.2$.

tion (19) the dimensionles volumetric entropy generation for the empty bed can be obtained, and is shown in Fig. 6 for $r = 0.2$. Similar results are also obtained for an empty duct by San et al. [21], Bejan [5] and Demirel and Al-Ali [6]. Comparison of Figs 5 and 6 indicate that outside the wall regions, the packed bed entropy generation rate is uniformly distributed which is an optimality criterion for transfer (heat and momentum) process [8, 9]. However, the wall region has an increasing effect on the uniformity of the entropy generation distribution with increasing H/d_p. This can also be seen in Fig. 7 where the entropy-generation distributions are displayed as functions of Y and H/d_p at $X = 10$ and $r = 0.2$.

The distortions at the wall regions are more visible in the graphical displays of the irreversibility distributions. Figure 8(a) and (b) shows the distributions of Be and ϕ, respectively, at $r = 0.2$. The entropy generation due to heat transfer dominates, and although the change of Be is very small, the wall effect on the distribution is clearly displayed. For the cooled wall, ϕ is relatively larger compared with the heated wall due to the less heat duty in this region.

Figure 9 shows the irreversibility distribution ratio at $H/d_p = 20$. The fluctuations are replaced by sharp changes due to scaling in ϕ in the wall regions.

4. Conclusions

The distributions of the volumetric entropy generation which is a measure of the irreversibilities for convection heat transfer have been determined and displayed graphically for fully developed velocity and temperature profiles in a large rectangular packed duct with asymmetric heating. The effects of the wall region (H/d_p) and the thermal boundary conditions (r) on the entropy generation have been displayed in a qualitative and quantitative manner. The velocity distortion near the walls destroys the uniformity in the entropy generation and irreversibility distributions in the packed bed. The distortions at the wall regions are larger for higher values of H/d_p (for which the wall region is narrower). In general, the entropy generation due to heat transfer dominates. The maps of the volumetric entropy generation reveal the thermodynamic behavior of the transfer system and lead to a better understanding and design of such systems with a required task.
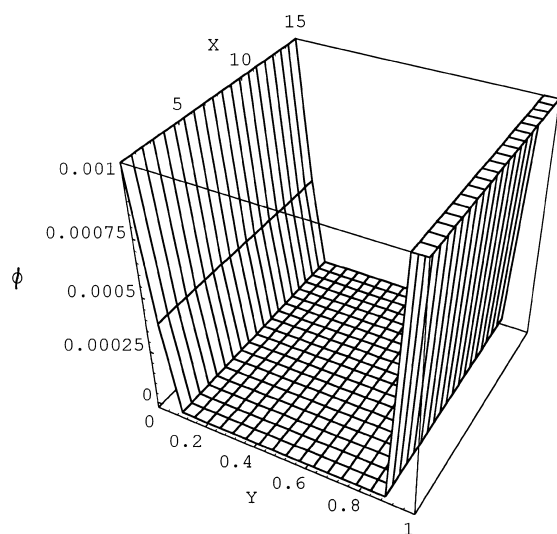
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Fig. 9. Irreversibility distribution ratio ϕ for $H/d_p = 20$, $Q_1 = 66$ W m^{-2}, $r = 0.2$, $Re_p = 77$, $\beta = 0.905$.

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